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INTUITIONISTIC FUZZY IDEALS

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ABSTRACT

In this paper, we introduce the concept of $\overline{Q_1}$-intuitionistic fuzzy ideal and $\overline{Q_2}$-intuitionistic fuzzy ideal and study their properties.

KEYWORDS: $\overline{Q}$-Intuitionistic Fuzzy Set, $\overline{Q}$-Intuitionistic Fuzzy Subsemiring, $\overline{Q_1}$-Intuitionistic Fuzzy Ideal & $\overline{Q_2}$-Intuitionistic Fuzzy Ideal.

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Original Article

INTRODUCTION

Zadeh [13] in 1965 introduced fuzzy sets once that many researchers explored on the generalizations of the notion of fuzzy sets and its application to many mathematical branches. Atanassov introduced intuitionistic fuzzy set that represent a genralization of the notion of fuzzy sets [1, 2]. A Solairaju and R. Nagarajan [8, 9, 10] have introduced and outlined a brand new algebraical structure known as$Q$-fuzzy subgroups. S. Hemalatha, et. al. [3] introduced the thought of Q-fuzzy subring of a ring and established some results. A study on anti Q-fuzzy subsemiring of a semiring has been introduced by Vanathi et. al. [11]. Some theorems in Q-intuitionistic fuzzy subsemiring of a semiring has been introduced by Vanathi et. al. [12]. O. Ratnabala Devi [7] introduced the thought of intuitionistic Q-fuzzy ideals of Near-rings. In this paper we introduce the thought of $\overline{Q_1}$-intuitionistic fuzzy ideals and $\overline{Q_2}$-intuitionistic fuzzy ideals. So far all Q-fuzzy subsets of rings, semirings, near-rings are studied with Q as a set only. In this paper, we introduce the thought of $\overline{Q}$-intuitionistic fuzzy subset of a semiring where $(Q,.)$ is a semigroup.

2. PRELIMINARIES

Definition 2.1

Let $(R,+,\cdot)$ be a semiring. Let $(Q,\cdot)$ be a semigroup. A map $A: R \times Q \to [0,1]$ is said to be a $\overline{Q}$-fuzzy subset of R.

Definition 2.2

Let $(R,+,,\cdot)$ be a semiring. Let $(Q,\cdot)$ be a semigroup. An $\overline{Q}$-intuitionistic fuzzy set is defined as $A = \{<(x,q), \mu_A(x,q), \nu_A(x,q) >/x \in R, q \in Q\}$ where the function $\mu_A: R \times Q \to [0,1]$ and $\nu_A: R \times Q \to [0,1]$ denote the degree of membership and degree of non-membership for each element $x \in R, q \in Q$ to the set A respectively and $0 \le \mu_A(x,q) + \nu_A(x,q) \le 1$, for each $x \in R, q \in Q$.

Definition 2.3

Let $(R,+,\cdot)$ be a semiring. A $\overline{Q}$-fuzzy subset A of R is said to be a $\overline{Q}$-fuzzy subsemiring of R if it satisfies the following conditions: [(i)]

- $\mu_A(x+y,q) \geq \min\{\mu_A(x,q),\mu_A(y,q)\}$
- $\mu_A(xy,q) \geq \min\{\mu_A(x,q),\mu_A(y,q)\}$
- $\mu_A(x,q,q_2) \geq \min\{\mu_A(x,q_1),\mu_A(x,q_2)\}, \forall x,y \in R$ and $q,q_1,q_2 \in Q$.

Definition 2.4

Let $(\mathrm{R},+,\cdot)$ be a semiring. A $\overline{\mathrm{Q}}$-fuzzy subset A of R is said to be an anti-$\overline{\mathrm{Q}}$-fuzzy subsemiring of R if it satisfies the following conditions:

- $\mu_A(x+y,q) \leq \max\{\mu_A(x,q),\mu_A(y,q)\}$
- $\mu_A(xy,q) \leq \max\{\mu_A(x,q),\mu_A(y,q)\}$
- $\mu_A(x,q,q_2) \leq \max\{\mu_A(x,q_1),\mu_A(x,q_2)\}$

Definition 2.5

Let $(\mathrm{R},+,\cdot)$ be a semiring. A $\overline{\mathrm{Q}}$-fuzzy subset A of R is said to be a $\overline{\mathrm{Q}}$-intuitionistic fuzzy subsemiring of R if it satisfies the following conditions:

- $\mu_A(x+y,q) \geq \min\{\mu_A(x,q),\mu_A(y,q)\}$
- $\mu_A(xy,q) \geq \min\{\mu_A(x,q),\mu_A(y,q)\}$
- $\mu_A(x,q.q_2) \geq \min\{\mu_A(x,q_1),\mu_A(x,q_2)\}$
- $\nu_A(x+y,q) \leq \max\{\nu_A(x,q),\nu_A(y,q)\}$
- $\nu_A(xy,q) \leq \max\{\nu_A(x,q),\nu_A(y,q)\}$
- $\nu_A(x,q.q_2) \leq \max\{\nu_A(x,q_1),\nu_A(x,q_2)\}$ for all $x,y \in R$ and $q,q_1,q_2 \in Q$.

3. $\overline{Q}$-INTUITIONISTIC FUZZY IDEAL: ($\overline{Q_2}$-INTUITIONISTIC FUZZY IDEAL)

Definition 3.1

Let $(\mathrm{R},+,\cdot)$ be a ring. A $\overline{\mathrm{Q}}_1$-intuitionistic fuzzy subset A of R is said to be $\overline{\mathrm{Q}_1}$-intuitionistic fuzzy ideal of R ($\overline{\mathrm{Q}_2}$-intuitionistic fuzzy ideal of R) for every $\mathrm{q} \in \mathrm{Q}$ if it satisfies the following conditions.

- $\mu_A(a+b,q) \geq \min\{\mu_A(a,q),\mu_A(b,q)\}$
- $\mu_A(ab,q) \geq \max\{\mu_A(a,q),\mu_A(b,q)\}$
- $\mu_A(a,q_1.q_2) \geq \min\{\mu_A(a,q_1),\mu_A(a,q_2)\}[\mu_A(x,q_1.q_2) \geq \max\{\mu_A(x,q_1),\mu_A(x,q_2)\}]$
- $\nu_A(a+b,q) \leq \max\{\nu_A(a,q),\nu_A(b,q)\}$
- $\nu_A(ab,q) \leq \min\{\nu_A(a,q),\nu_A(b,q)\}$
- $\nu_A(a,q_1.q_2) \leq \max\{\nu_A(a,q_1),\nu_A(a,q_2)\}[\nu_A(a,q_1.q_2) \leq \min\{\nu_A(a,q_1),\nu_A(a,q_2)\}]$

for all $a,b \in R$ and $q_1,q_2 \in Q$.

Definition 3.2

$\overline{Q_2}$-intuitionistic fuzzy ideal $\Rightarrow$ $\overline{Q_1}$-intuitionistic fuzzy ideal. But the converse is not true. It is shown by the following example. Let R be the set of real numbers and Q be the set of rational numbers. Then $(R,+,\cdot)$ is a semiring and $(Q,\cdot)$ is a semigroup. We define $\overline{Q}$-intuitionistic fuzzy set $A =< \mu_A, \nu_A >$ in R as follows:

$$\mu_A(a,q) = \begin{cases} 1 & \text{if } a \in R \text{ and } q \in Q \\ 0 & \text{otherwise} \end{cases}$$

and

$$\nu_A(a,q) = \begin{cases} 0 & \text{if } a \in R \text{ and } q \in Q \\ 1 & \text{otherwise} \end{cases}$$

Clearly, $A =< \mu_A, \nu_A >$ is a $\overline{Q_1}$-intuitionistic fuzzy ideal, but it is not a $\overline{Q_2}$-intuitionistic fuzzy ideal since $\mu_A(1,2\sqrt{2}) \ngeq max\{\mu_A(1,2), \mu_A(1,\sqrt{2})$.

Definition 3.3

Let θ be a mapping from semiring X to semiring Y. For any intuitionistic fuzzy set $B =< \mu_B, \nu_B >$ in Y, we define a new intuitionistic fuzzy set denoted as $\theta^{-1}(B) =< \mu_{\theta^{-1}(B)}, \nu_{\theta^{-1}(B)} >$ in X where $\mu_{\theta^{-1}B}(a,q) = \mu_B(\theta(a),q)$ and $\nu_{\theta^{-1}(B)}(a,q) = \nu_B(\theta(a),q)$ for all $a \in X, q \in Q$. For any intuitionistic fuzzy set $A =< \mu_A, \nu_A >$ in X, we define $\theta(A)$ denoted as $\theta(A) =< \mu_{\theta(A)}, \nu_{\theta(A)} >$ in Y by

$$\mu_{\theta(A)}(b,q) = \begin{cases} \sup\limits_{a\in\theta^{-1}(b)} \mu_A(a,q), & \text{if } \theta^{-1}(b) \neq \phi \\ 0, & \text{otherwise} \end{cases}$$

$$\nu_{\theta(A)}(b,q) = \begin{cases} \inf\limits_{a\in\theta^{-1}(\mathrm{b})} \nu_A(a,q), & \text{if } \theta^{-1}(b) \neq \phi \\ 1, & \text{otherwise} \end{cases}$$

Theorem 3.4

Let $f: R \to R'$ be surjective ring homomorphism. Then if A is $\overline{Q_1}$-intuitionistic fuzzy ideal in R($\overline{Q_2}$-intuitionistic fuzzy ideal in R) then $f(A)$ is $\overline{Q_1}$-intuitionistic fuzzy ideal in R'. ($\overline{Q_2}$-intuitionistic fuzzy ideal in R'.)

Proof

Let A be $\overline{Q_1}$-intuitionistic fuzzy ideal in R. Let $a_1, a_2 \in R, b_1, b_2 \in R'$ and $q, q_1, q_2 \in Q$.

$$\begin{aligned} \text{Now } \mu_{f(A)}(b_1+b_2,q) &= \sup\{\mu_A(a_1-a_2,q)/a_1-a_2 \in f^{-1}(b_1-b_2)\} \\ &\geq \sup\{\min\{\mu_A(a_1,q),\mu_A(a_2,q)\}/a_1 \in f^{-1}(b_1), a_2 \in f^{-1}(b_2)\} \\ &= \min\{\sup\{\mu_A(a_1,q)/a_1 \in f^{-1}(b_1)\}, \sup\{\mu_A(a_2,q)/a_2 \in f^{-1}(b_2)\}\} \\ &= \min\{\mu_{f(A)}(b_1,q), \mu_{f(A)}(b_2,q)\} \end{aligned}$$

$$\begin{aligned} \text{Also } \mu_{f(A)}(b_1b_2,q) &= \sup\{\mu_A(a_1a_2,q)/a_1a_2 \in f^{-1}(b_1b_2)\} \\ &\geq \sup\{\max\{\mu_A(a_1,q),\mu_A(a_2,q)\}/a_1 \in f^{-1}(b_1), a_2 \in f^{-1}(b_2)\} \end{aligned}$$

$$= \max\{\sup\{\mu_A(a_1,q)/a_1 \in f^{-1}(b_1)\}, \sup\{\mu_A(a_2,q)/a_2 \in f^{-1}(b_2)\}\}$$

$$= \max\{\mu_{f(A)}(b_1,q), \mu_{f(A)}(b_2,q)\}$$

and $\mu_{f(A)}(b_1, q_1.q_2) = \sup\{\mu_A(a_1,q_1.q_2)/a_1 \in f^{-1}(b_1)\}$

$$\geq \sup\{\min\{\mu_A(a_1,q_1), \mu_A(a_1,q_2)\}/a_1 \in f^{-1}(b_1)\}$$

$$= \min\{\sup\{\mu_A(a_1,q_1)/a_1 \in f^{-1}(b_1)\}, \sup\{\mu_A(a_1,q_2)/a_1 \in f^{-1}(b_1)\}\}$$

$$= \min\{\mu_{f(A)}(b_1,q_1), \mu_{f(A)}(b_1,q_2)\}$$

Now $\nu_{f(A)}(b_1+b_2,q) = \inf\{\nu_A(a_1-a_2,q)/a_1-a_2 \in f^{-1}(b_1-b_2)\}$

$$\leq \inf\{\max\{\nu_A(a_1,q), \nu_A(a_2,q)\}/a_1 \in f^{-1}(b_1), a_2 \in f^{-1}(b_2)\}$$

$$= \max\{\inf\{\nu_A(a_1,q)/a_1 \in f^{-1}(b_1)\}, \inf\{\nu_A(a_2,q)/a_2 \in f^{-1}(b_2)\}\}$$

$$= \max\{\nu_{f(A)}(b_1,q), \nu_{f(A)}(b_2,q)\}$$

Also $\nu_{f(A)}(b_1b_2,q) = \inf\{\nu_A(a_1a_2,q)/a_1a_2 \in f^{-1}(b_1b_2)\}$

$$\leq \inf\{\min\{\nu_A(a_1,q), \nu_A(a_2,q)\}/a_1 \in f^{-1}(b_{1_1}), a_2 \in f^{-1}(b_2)\}$$

$$= \min\{\inf\{\nu_A(a_1,q)/a_1 \in f^{-1}(b_1)\}, \inf\{\nu_A(a_2,q)/a_2 \in f^{-1}(b_2)\}\}$$

$$= \min\{\nu_{f(A)}(b_1,q), \nu_{f(A)}(b_2,q)\}$$

and $\nu_{f(A)}(b_1,q_1.q_2) = \inf\{\nu_A(a_1,q_1.q_2)/a_1 \in f^{-1}(b_1)\}$

$$\leq \inf\{\max\{\nu_A(a_1,q_1), \nu_A(a_1,q_2)\}/a_1 \in f^{-1}(b_1)\}$$

$$= \max\{\inf\{\nu_A(a_1,q_1)/a_1 \in f^{-1}(b_1)\}, \inf\{\nu_A(a_1,q_2)/a_1 \in f^{-1}(b_2)\}\}$$

$$= \max\{\nu_{f(A)}(b_1,q_1), \nu_{f(A)}(b_1,q_2)\}$$

Hence $f(A)$ is $\overline{Q_1}$-intuitionistic fuzzy ideal of semiring R.

Theorem 3.5

Let $\mathrm{f}: \mathrm{R} \to \mathrm{R}'$ be surjective ring homomorphism. If B is a $\overline{\mathrm{Q}_1}$-intuitionistic fuzzy ($\overline{\mathrm{Q}_2}$-intuitionistic fuzzy ideal) in R' then $\mathrm{f}^{-1}(\mathrm{B})$ is $\overline{\mathrm{Q}_1}$-intuitionistic fuzzy ideal ($\overline{\mathrm{Q}_2}$-intuitionistic fuzzy ideal) in R.

Proof

Let B be a $\overline{Q_1}$-intuitionistic fuzzy ideal in R'. Let $a_1, a_2 \in R$ and $q, q_1, q_2 \in Q$.

Now $\mu_{f^{-1}(B)}(a_1+a_2,q) = \mu_B(f(a_1+a_2),q) = \mu_B(f(a_1)+f(a_2),q)$

$$\geq \min\{\mu_B(f(a_1),q), \mu_B(f(a_2),q) = \min\{\mu_{f^{-1}(B)}(a_1,q), \mu_{f^{-1}(B)}(a_2,q)\}$$

Also $\mu_{f^{-1}(B)}(a_1a_2,q) = \mu_B(f(a_1a_2),q) = \mu_B(f(a_1)f(a_2),q)$

$$\geq \min\{\mu_B(f(a_1),q), \mu_B(f(a_2),q) = \min\{\mu_{f^{-1}(B)}(a_1,q), \mu_{f^{-1}(B)}(a_2,q)\}$$

and $\mu_{f^{-1}(B)}(a_1, q_1.q_2) = \mu_B(f(a_1), q_1.q_2) \geq \min\{\mu_B(f(a_1), q_1), \mu_B(f(a_1), q_2)$

$$= \min\{\mu_{f^{-1}(B)}(a_1, q_1), \mu_{f^{-1}(B)}(a_2, q_2)\}$$

Now $\nu_{f^{-1}(B)}(a_1 + a_2, q) = \nu_B(f(a_1 + a_2), q) = \nu_B(f(a_1) + f(a_2), q)$

$$\leq \max\{\nu_B(f(a_1), q), \nu_B(f(a_2), q) = \max\{\nu_{f^{-1}(B)}(a_1, q), \nu_{f^{-1}(B)}(a_2, q)\}$$

$\nu_{f^{-1}(B)}(a_1 a_2, q) = \nu_B(f(a_1 a_2), q) = \nu_B(f(a_1)f(a_2), q)$

$$\leq \max\{\nu_B(f(a_1), q), \nu_B(f(a_2), q) = \max\{\nu_{f^{-1}(B)}(a_1, q), \nu_{f^{-1}(B)}(a_2, q)\}$$

and $\nu_{f^{-1}(B)}(a_1, q_1.q_2) = \nu_B(f(a_1), q_1.q_2) \leq \max\{\nu_B(f(a_1), q_1), \nu_B(f(a_1), q_2)$

$$= \max\{\nu_{f^{-1}(B)}(a_1, q_1), \nu_{f^{-1}(B)}(a_2, q_2)\}$$

Hence $f^{-1}(B)$ is a $\overline{Q_1}$-intuitionistic fuzzy ideal of R.

Definition 3.6

Let $(\mathrm{R}, +, \cdot)$ be a semiring, $(\mathrm{Q}, \cdot)$ be a semigroup and $\mu: \mathrm{R} \times \mathrm{Q} \to [0,1]$ be a $\overline{\mathrm{Q}}$-fuzzy set. An upper level set $\mathrm{U_q}(\mu; \mathrm{t})$ of a $\overline{\mathrm{Q}}$-fuzzy set μ is defined as for $\mathrm{q} \in \mathrm{Q}, \mathrm{U_q}(\mu; \mathrm{t}) = \{\mathrm{a} \in \mathrm{R}/\mu(\mathrm{a}, \mathrm{q}) \geq \mathrm{t}\}$ and a lower level set $\mathrm{L_q}(\mu; \mathrm{t})$ of a $\overline{\mathrm{Q}}$-fuzzy set μ is defined as for $\mathrm{q} \in \mathrm{Q}, \mathrm{L_q}(\mu; \mathrm{t}) = \{\mathrm{a} \in \mathrm{R}/\mu(\mathrm{a}, \mathrm{q}) \leq \mathrm{t}\}$, for all $\mathrm{t} \in [0,1]$.

Definition 3.7

The intersection of all upper level sets $\mathrm{U_q}(\mu; \mathrm{t})$ is denoted by $\mathrm{U}(\mu; \mathrm{t})$i.e., $\mathrm{U}(\mu; \mathrm{t}) = \bigcap_{\mathrm{q} \in \mathrm{Q}} \mathrm{U_q}(\mu; \mathrm{t})$

The intersection of all lower level sets $L_q(\mu; t)$ is denoted by $L(\mu; t) i.e., L(\mu; t) = \bigcap_{q \in Q} L_q(\mu; t)$

Definition 3.8

Let $(\mathrm{R}, +, \cdot)$ be a semiring, $(\mathrm{Q}, \cdot)$ be a semigroup and $\mu: \mathrm{R} \times \mathrm{Q} \to [0,1]$ be a $\overline{\mathrm{Q}}$-fuzzy set. An upper level set $\overline{\mathrm{U_a}}(\mu; \mathrm{t})$ for a given $\mathrm{a} \in \mathrm{R}$ of a $\overline{\mathrm{Q}}$-fuzzy set μ is defined as $\overline{\mathrm{U_a}}(\mu; \mathrm{t}) = \{\mathrm{q} \in \mathrm{Q}/\mu(\mathrm{a}, \mathrm{q}) \geq \mathrm{t}\}$ and a lower level set $\overline{\mathrm{L_a}}(\mu; \mathrm{t})$ for a given $\mathrm{a} \in \mathrm{R}$ of a $\overline{\mathrm{Q}}$-fuzzy set μ is defined as$\overline{\mathrm{L_a}}(\mu; \mathrm{t}) = \{\mathrm{q} \in \mathrm{Q}/\mu(\mathrm{a}, \mathrm{q}) \leq \mathrm{t}\}$.

Definition 3.9

The intersection of all upper level sets $\overline{\mathrm{U_a}}(\mu; \mathrm{t})$ is denoted by $\overline{\mathrm{U}}(\mu; \mathrm{t})$i.e., $\overline{\mathrm{U}}(\mu; \mathrm{t}) = \bigcap_{\mathrm{a} \in \mathrm{R}} \overline{\mathrm{U_a}}(\mu; \mathrm{t})$

The intersection of all lower level sets $\overline{L_a}(\mu; t)$ is denoted by $\overline{L}(\mu; t) i.e., \overline{L}(\mu; t) = \bigcap_{a \in R} \overline{L_a}(\mu; t)$

Theorem 3.10

If A is a $\overline{\mathrm{Q_1}}$-intuitionistic fuzzy ideal ($\overline{\mathrm{Q_2}}$-intuitionistic fuzzy ideal) of a semiring $(\mathrm{R}, +, \cdot)$, then $\mathrm{U_q}(\mu_\mathrm{A}; \mathrm{t})$ and $\mathrm{L_q}(\nu_\mathrm{A}; \mathrm{t})$ are ideals of R for every $\mathrm{q} \in \mathrm{Q}$.

Proof

Given A is a $\overline{Q_1}$-intuitionistic fuzzy ideal of a semiring $(R, +, \cdot)$. We have to prove that $U_q(\mu_A; t)$ and $L_q(\nu_A; t)$ are ideals of R.

Let $q \in Q, a, b \in U_q(\mu_A; t)$ and $t \in [0,1]$ then $\mu_A(a, q) \geq t$ and $\mu_A(b, q) \geq t$

Now $\mu_A(a + b, q) \geq \min\{\mu_A(a, q), \mu_A(b, q)\} \geq \min\{t, t\} = t \Rightarrow a + b \in U_q(\mu_A; t)$

Let $q \in Q, a \in U_q(\mu_A; t)$ and b$\in R$.

$\mu_A(\text{ab}, q) \geq \max\{\mu_A(a, q), \mu_A(b, q)\} \geq \min\{a, q\} \geq t \Rightarrow ab \in U_q(\mu_A; t)$

Thus $U_q(\mu_A; t)$ is an ideal of R.

$|||^{rly}$ We can prove that $L_q(\nu_A; t)$ is an ideal of R.

Corollary 3.11

If A is $\overline{Q_1}$-intuitionistic fuzzy ideal($\overline{Q_2}$-intuitionistic fuzzy ideal) of a semiring $(R, +, \cdot)$ then $U(\mu_A; t) = \bigcap_{q \in Q} U_q(\mu_A; t)$ and $L(\nu_A; t) = \bigcap_{q \in Q} L_q(\mu_A; t)$ are ideals of R.

Theorem 3.12

If A is a $\overline{Q_1}$-intuitionistic fuzzy ideal ($\overline{Q_2}$-intuitionistic fuzzy ideal) of a semiring R, then $\overline{U_a}(\mu_A; t)$ and $\overline{L_a}(\nu_A; t)$ are subsemigroup (ideals) of Q for every $a \in R$.

Proof

Given A is a $\overline{Q_1}$-intuitionistic fuzzy ideal of a semiring R. We have to prove that $\overline{U_a}(\mu_A; t)$ and $\overline{L_a}(\nu_A; t)$ are subsemigroup of Q.

Let $a \in R, q_1, q_2 \in \overline{U_a}(\mu_A; t)$ and $t \in [0,1]$ then $\mu_A(a, q_1) \geq t$ and $\mu_A(a, q_2) \geq t$.

Now $\mu_A(a, q_1 q_2) \geq \min\{\mu_A(a, q_1), \mu_A(a, q_2)\} \geq \min\{t, t\} = t \Rightarrow q_1 q_2 \in \overline{U_a}(\mu_A; t)$

Let $a \in R, q_1, q_2 \in \overline{L_a}(\nu_A; t)$ and $t \in [0,1]$ then $\nu_A(a, q_1) \leq t, \nu_A(a, q_2) \leq t$.

Now $\nu_A(a, q_1 q_2) \leq \max\{\nu_A(a, q_1), \nu_A(a, q_2)\} \leq \max\{t, t\} = t \Rightarrow q_1 q_2 \in \overline{L_a}(\nu_A; t)$

Hence $\overline{U_a}(\mu_A; t)$ and $\overline{L_a}(\nu_A; t)$ are subsemigroup of Q.

Corollary 3.13

If A is a $\overline{Q_1}$-intuitionistic fuzzy ideal of a semiring R ($\overline{Q_2}$-intuitionistic fuzzy ideal) then $\overline{U}(\mu_A; t) = \bigcap_{a \in R} \overline{U_a}(\mu_A; t)$ and $\overline{L}(\nu_A; t) = \bigcap_{a \in R} \overline{L_a}(\nu; t)$ are subsemigroup (ideals) of Q.

Theorem 3.14

If A is a $\overline{Q_1}$-intuitionistic fuzzy ideal of a semiring R if and only if for given $q \in Q$ and $a \in R, U_q(\mu_A; t), L_q(\nu_A; t)$ are ideals of R and $\overline{U_a}(\mu_A; t), \overline{L_a}(\nu_A; t)$ are subsemigroup of Q.

Proof

Consider A is a $\overline{Q_1}$-intuitionistic fuzzy ideal of R. Then for $q \in Q$, by Theorem 3.10, $U_q(\mu_A; t)$ and $L_q(\nu; t)$ are ideals of R. For $x \in R$, by Theorem 3.12, $\overline{U_a}(\mu_A; t)$ and $\overline{L_a}(\nu_A; t)$ are subsemigroup of Q.

Conversely, let $U_q(\mu_A;t)$ and $L_q(\nu_A;t)$ be ideals of R and $\overline{U_a}(\mu_A;t)$ and $\overline{L_a}(\nu_A;t)$ be subsemigroup of Q. We have to prove A is a $\overline{Q_1}$-intuitionistic fuzzy ideal of R. Let a, b $\in R, q \in Q$ and $t_1 = \min\{\mu_A(a,q), \mu_A(b,q)\}$ and $t_2 = \max\{\nu_A(a,q), \nu_A(b,q)\}$. Then $\mu_A(a,q) \geq t_1, \mu_A(b,q) \geq t_1$ and $\nu_A(a,q) \leq t_2, \nu_A(b,q) \leq t_2 \Rightarrow a,b \in U_q(\mu_A;t_1)$ and $a,b \in L_q(\nu_A;t_2)$

$\Rightarrow a+b \in U_q(\mu_A;t_1)$ and $a+b \in L_q(\nu_A;t_2)[\because U_q(\mu_A;t_1)$ and $L_q(\nu_A;t_1)$ are ideals of R].

So $\mu_A(a+b,q) \geq t_1 = \min\{\mu_A(a,q), \mu_A(b,q)\}$

and $\nu_A(a+b,q) \leq t_2 = \max\{\nu_A(a,q), \nu_A(b,q)\}$

Now let $t_3 = \max\{\mu_A(a,q), \mu_A(b,q)\}$, say $t_3 = \mu_A(a,q)$

Since $\mu_A(a,q) + \nu_A(a,q) \leq 1, \nu_A(a,q) \leq 1 - t_3 = t_4$

Thus $a \in U_q(\mu_A;t_3)$

Since $U_q(\mu_A;t_3)$ is an ideal, for $b \in R$, we get $ab \in U_q(\mu_A;t_3)$

$\therefore \mu_A(ab,q) \geq t_3 = \max\{\mu_A(a,q), \mu_A(b,q)\}$

$|||^{rly}$ we can show that $\nu_A(ab,q) \leq \min\{\nu_A(a,q), \nu_A(b,q)\}$

Now let us show that $\mu_A(a,q_1q_2) \geq \min\{\mu_A(a,q_1), \mu_A(a,q_2)\}$ for any $a \in R, q_1, q_2 \in Q$

Let $t_5 = \min\{\mu_A(a,q_1), \mu_A(a,q_2)\}$ then $\mu_A(a,q_1) \geq t_5, \mu_A(a,q_2) \geq t_5$

$\Rightarrow q_1, q_2 \in \overline{U_a}(\mu_A;t_5)$

$\Rightarrow q_1q_2 \in \overline{U_a}(\mu_A;t_5)[\because U_a(\mu_A;t_5)$ is a subsemigroup of $Q]$

$\therefore \mu_A(a,q_1.q_2) \geq t_5 = \min\{\mu_A(a,q_1), \mu_A(a,q_2)\}$

$|||^{rly}$ we can show that $\nu_A(a,q_1.q_2) \leq \max\{\nu_A(a,q_1), \nu_A(a,q_2)\}$.

Hence A is a $\overline{Q_1}$-intuitionistic fuzzy ideal of R.

Theorem 3.15

A is a $\overline{Q_2}$-intuitionistic fuzzy ideal of a semiring $(R,+,\cdot)$, if and only if for given $q \in Q, x \in R, U_q(\mu_A;t)$ and $L_q(\nu_A;t)$ are ideals of R and $\overline{U_a}(\mu_A;t), \overline{L_a}(\nu_A;t)$ are ideals of Q.

Proof

Consider A is a $\overline{Q_2}$-intuitionistic fuzzy ideal of R. Then for $q \in Q$, by Theorem 3.10, $U_q(\mu_A;t)$ and $L_q(\nu;t)$ are ideals of R. For $a \in R$, by Theorem 3.12, $\overline{U_a}(\mu_A;t)$ and $\overline{L_a}(\nu_A;t)$ are ideals of Q.

Conversely, let $U_q(\mu_A;t)$ and $L_q(\nu_A;t)$ be ideals of R and $\overline{U_a}(\mu_A;t)$ and $\overline{L_a}(\nu_A;t)$ be ideals of Q. We have to prove A is a $\overline{Q_2}$-intuitionistic fuzzy ideal of R.

Let $a,b \in R, q \in Q$ and $t_1 = \min\{\mu_A(a,q), \mu_A(b,q)\}$ and $t_2 = \max\{\nu_A(a,q), \nu_A(b,q)\}$

Then $\mu_A(a,q) \geq t_1, \mu_A(b,q) \geq t_1$ and $\nu_A(a,q) \leq t_2, \nu_A(b,q) \leq t_2$

$\Rightarrow a,b \in U_q(\mu_A;t_1)$ and $a,b \in L_q(\nu_A;t_2)$

$\Rightarrow a+b \in U_q(\mu_A;t_1)$ and $a+b \in L_q(\nu_A;t_2)[\because U_q(\mu_A;t_1)$ and $L_q(\nu_A;t_1)$ are ideals of R].

So $\mu_A(a+b,q) \geq t_1 = \min\{\mu_A(a,q), \mu_A(b,q)\}$

and $\nu_A(a+b,q) \leq t_2 = \max\{\nu_A(a,q), \nu_A(b,q)\}$

Now let $t_3 = \max\{\mu_A(a,q), \mu_A(b,q)\}$, say $t_3 = \mu_A(a,q)$

Since $\mu_A(a,q) + \nu_A(a,q) \leq 1, \nu_A(a,q) \leq 1 - t_3 = t_4$

Thus $a \in U_q(\mu_A;t_3)$

Since $U_q(\mu_A;t_3)$ is an ideal of R, for $b \in R$, we get $ab \in U_q(\mu_A;t_3)$

$\therefore \mu_A(ab,q) \geq t_3 = \max\{\mu_A(a,q), \mu_A(b,q)\}$

$|||^{rly}$ we can show that, $\nu_A(ab,q) \leq \min\{\nu_A(a,q), \nu_A(b,q)\}$

Now let us show that $\mu_A(a,q_1.q_2) \geq \max\{\mu_A(a,q_1), \mu_A(\mathrm{a},q_2)\}$ for any $x \in R, q_1,q_2 \in Q$.

Let $t_5 = \max\{\mu_A(a,q_1), \mu_A(a,q_2)\}$, say $t_5 = \mu_A(a,q_1)$

Thus $q_1 \in \overline{U_q}(\mu_A;t_5)$. Since $U_q(\mu_A;t_5)$ is an ideal of Q, for $q_2 \in Q$, we get $q_1.q_2 \in \overline{U_q}(\mu_A;t_5)$

$\therefore \mu_A(a,q_1.q_2) \geq t_5 = \max\{\mu_A(a,q_1), \mu_A(a,q_2)\}$

$|||^{rly}$ we can show that $\nu_A(a,q_1.q_2) \leq \min\{\nu_A(a,q_1), \nu_A(a,q_2)\}$.

4. CONCLUSIONS

In this paper $\overline{\mathrm{Q}_1}$-intuitionistic fuzzy ideal and $\overline{\mathrm{Q}_2}$-intuitionistic fuzzy ideal were introduced and their basic algebraic properties were studied. The upper level sets and lower level sets of a $\overline{\mathrm{Q}}$- fuzzy setswere defined and their basic structures were explored. A similar approach can be explained for manyrings.

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